

A PEDAGOGICAL LEGACY? ABACUS TRADITION AND SURPRISING CONNECTIONS. FROM THE *LIBER ABACI* (1202) TO THE ECONOMIC EDUCATION IN EARLY MODERN PERIOD

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ABSTRACT

The *abacus tradition* had a primary educational role between the late Middle Ages and the Early Modern Era. This practical pedagogy was addressed especially to the merchant and artisan classes. It developed especially in Italy through the presence of private teachers, whose didactics was based initially and primarily on Fibonacci's works. Commercial mathematics, practical geometry, accounting systems and financial rules found in the abacus environment their incubation and development. The main objective of this article is to demonstrate some (unexpected and surprising) connections between the abacus tradition and the pedagogical evolution of the Modern Period.

KEYWORDS: Economic History, Medieval History, History of Early Modern Period, History of Education, History of Economic Thought, Renaissance Studies.

¿UN LEGADO PEDAGÓGICO? LA TRADICIÓN DEL ÁBACO Y SUS SORPRENDENTES CONEXIONES. DEL *LIBER ABACI* (1202) A LA EDUCACIÓN ECONÓMICA EN LA TEMPRANA EDAD MODERNA

RESUMEN

La tradición del ábaco desempeñó un papel educativo primordial entre la Baja Edad Media y la primera Edad Moderna. Esta pedagogía práctica se dirigía especialmente a las clases de mercaderes y artesanos. Se desarrolló sobre todo en Italia gracias a la presencia de maestros privados, cuya didáctica se basaba inicialmente y principalmente en las obras de Fibonacci. Las matemáticas comerciales, la geometría práctica, los sistemas contables y las reglas financieras encontraron en el entorno del ábaco su incubación y desarrollo. El principal objetivo de este artículo es demostrar algunas conexiones (inesperadas y sorprendentes) entre la tradición del ábaco y la evolución pedagógica de la Edad Moderna.

PALABRAS CLAVE: Historia Económica, Historia Medieval, Historia de la Edad Moderna, Historia de la Pedagogía, Historia del Pensamiento Económico, Estudios Renacentistas.

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0. INTRODUCTION

The *abacus tradition* had a primary educational role between the late Middle Ages and the Early Modern Era. This practical pedagogy was addressed especially to the merchant and artisan classes. It developed especially in central-northern Italy through the presence of private teachers (Grendler, 1989). In their schools they offered a modular didactics almost exclusively in the vernacular language (Fanci&Toti Rigatelli, 1989). It was calibrated on arithmetical operations with integers, the calculation of fractions and problems relating to the system of weights, measures and coins with their pure and impure metallic alloys as well as exchange rates, interest rates and discount rates (Patriarca, 2021).

This *science of numbers* undoubtedly owes much to the fundamental contribution of Leonardo Fibonacci (Maccagni, 1988: 91-113) through his celebrated *Liber abaci* (1202; 2019, 2020), but also thanks to the extremely wide circulation in Europe of his *Practica geometriae* published between 1220 and 1221. In this work, with astonishing linguistic and didactic skills, starting from Euclidean elements (Folkerts, 2004: 93-113) and the speculations of Islamic mathematics (Rebstock, 1994: 91-115), he deeply analyses eight “distinctiones” (distinctions). In them he deals with the division of regular surfaces, the determination of the volume of various solids, the calculation of the areas of plane figures, the extraction of square and cubic roots, as well as some problems for the determination of heights and distances.

It was followed –not without success– by two other books of central importance: the *liber quadratorum* (1225) (Picutti, 1979: 195-239) and the (undated) *Flos super solutionibus quarundam questionibus ad numerum et ad geometriam, vel ad utrumque pertinentium* (Picutti, 1983:293-387). In them, the Latin appears scientifically purified and essential. In his works he develops not only the most stringent mathematical questions of his time but also commercial arithmetic, accounting and problems of exchange, weight and measurement. Fibonacci’s style and contents initiated a pedagogical tradition of masters with their private educational institutions aimed especially at the merchant class (Ulivi, 2011: 247-288). These *abacus masters* were also central to the dissemination and vulgarisation of Euclidean mathematics and the Arabic scientific tradition. The contribution of other important mathematicians such as Jordanus de Nemore (Grant, 2005: 294-295) and Johannes Campanus de Novara (Paravicini Bagliani, 1973: 98-129) appears mediated by their teachings.

Jordanus de Nemore possesses –in a direct and simplified Latin– distinguished teaching skills with completely innovative methodologies. His texts can be considered the first manuals on arithmetic and geometry. In them, he expounds detailed propositions on the relationship between arcs and plane segments in circles, the division of straight lines, and the relationship between angles and sides. The

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study of these topics reaches its peak in *De numeris datis, De triangulis* as well as in *Demonstratio de plana spera* (Thomson, 1978).

Johannes Campano de Novara's Latin edition of Euclid's *Elementa geometriae* is supported by a detailed commentary with clearer and more accessible proofs (Campanus Novariensis, Siglo XIII). This pedagogical simplification with precise explanations also allows a wider diffusion. Through alternative forms and complementary theorems (Cantor, 1913: 100-106), he introduces an innovative system for calculating the angles of the pentagon (Murdoch, 1968: 67-94). This cultural vibrancy, made even more alive by the intellectual flowering of the universities, is joined –albeit in a slow and articulate manner– with the surprising results of the scientific revolution underway in those years between Oxford and Paris (Maier, 1952; Duhem, 1959). To the *Calculators of Merton College* and scholars such as Buridan, Oresme and Themon, medieval mathematics and physics are highly indebted for significant contributions in kinematics, astronomy and optics, as well as the study of speed and motion (Clagett, 1959; Hugonnard-Roche, 1973; Grant, 1981; Crombie, 1990; Dudley Sylla, 1991; Courtenay, 2005).

Franciscan Luca Pacioli is credited with codifying –through the *Summa de arithmetica, geometria, proportioni e proportionalità* (1494)– a fundamental part of the abacus tradition. His contribution to the science of accounting and applied mathematics is undoubtedly so great that –according to the American historian Henry Rand Hatfield– «it is seldom the case that a first book on a subject has so dominated its literature as was the case with Pacioli's *De Computis et Scripturis*. It is nearly true that for a hundred years the texts appearing in England, France, Germany Italy and the Low Countries were at the best revisions of Pacioli, at the worst servile transcriptions without even the courtesy of referring to the original author» (Rand Hatfield, 1924: 245).

Yet Pacioli summarizes and completes a tradition that is more than a hundred years old and from which further developments would begin until the Renaissance *abacus treatises*, in which the step towards modern mathematics is visible due to the contributions of Scipione del Ferro (1465-1526), Bartolomeo Zamberti (1473-1543), Francesco Maurolico (1494-1575), Nicolò Tartaglia (c. 1499-1557), Gerolamo Cardano (1501-1576) and his pupil Ludovico Ferrari (1522-1565).

The main objective of this essay is to demonstrate the (unexpected) connections between the Medieval abacus tradition and the pedagogical evolution of the Early Modern Age, which –silently– borrowed from the *abacus schools* the methodology and technique for learning the mathematical, physical and actuarial sciences (Gamba & Montebelli, 1987: 169-202). Some surprising correlations will come to light and may be a stimulus for further monographic investigations.

1. AN EXPERIMENTAL PEDAGOGY

The pedagogical novelty and the modular structure of the *abacus schools* extends along the lines of international trade. Not infrequently in the main European cities and ports, *abacus teachers* are also found for the human and professional



education of the younger generations. The success of their methodology is due to their practicality (Høyrup, 2009: 47-75). They fill a pedagogical gap and are aimed at those who had a greater propensity for creativity. In today's language, they would be integrated into the *theory of multiple intelligences* (Gardner, 1987) or in the *experiential-creative sub-theories* (Sternberg, 1997: 43-53).

They are fundamental to a technical and professional evolution in the late Middle Ages. The *abacus schools* are, in fact, a harbinger of procedural innovations not only in the purely accounting field but also in the mathematical implications of constructions, nautics and military infrastructures. They are the essential link between science and practice, transferring their advances to the cultural peripheries. It is interesting to note how their role is, (not only) from an epistemological point of view, a milestone in the history of pedagogy and vocational training (Patriarca, 2022). Their existence opens a veil of shadow that allows, finally, a greater consideration of some (hitherto marginalised) social groups. Their pupils and students were more interested in a practical effectiveness of their knowledge than in the theoretical speculation.

Leonardo da Vinci (1452-1519) in the *Trattato della pittura* perfectly captures its meaning with captivating and critical clarity, referring, perhaps not by chance, to the naval language (Musarra, 2021). In this regard, he argues that: «Those who fall in love with practice without science are like helmsmen who enter a ship without rudder and compass, who never have certainty where they are going. Always practice must be built on top of good theory, of which perspective is the guide and door, and without this nothing is done well» (Vinci, 1996: 45)¹.

The childhood of Leonardo, a *polymath* who certainly marks the transition to the Renaissance, is an interesting starting point for understanding the fundamental influence of the *abacus schools* for the subsequent flourishing of the arts. At the beginning of his famous *Vita di Leonardo*, Giorgio Vasari, in presenting his multifaceted and vivacious personality, writes

in the erudition and principles of letters would have made great profit, if he had not been so varied and unstable. Therefore he set himself to learn many things; and having begun them, he then abandoned them. Behold, in the abacus, he in a few months that he attended there, made so much acquisition, that by continually moving doubts and difficulties to the master who taught him, well often confounded him (Vasari, 1996: IV)².

¹ «Quelli che si innamorano della pratica senza la scienza, sono come i nocchieri che entrano in naviglio senza timone e bussola, che mai hanno certezza dove si vadano. Sempre la pratica deve essere edificata sopra la buona teorica, della quale la prospettiva è guida e porta, e senza questa nulla si fa bene».

² «nella erudizione e principj delle lettere avrebbe fatto profitto grande, se egli non fosse stato tanto vario e instabile. Perciocchè egli si mise a imparare molte cose; e incominciate, poi l'abbandonava. Ecco, nell'abbaco, egli in pochi mesi ch'è v'attese, fece tanto acquisto, che movendo di continuo dubbj e difficoltà al maestro che gl'insegnava, bene spesso lo confondeva».



This biographical reference –even if taken with a certain distance due to an approach bordering on hagiography– allows to understand how the abacus tradition was a pillar in the Renaissance pedagogical conception. The question that might arise at this point is whether there were some (unexpected) interconnections between *abacus legacy* and the pedagogical innovations that were to develop in the Early Modern Age with a whole series of far from minor consequences. Especially in the didactics of mathematics, this subtle link seems not only to be present but to have been a kind of leaven for their structuring.

2. FROM THE COLLEGE OF GUYENNE TO THE JESUITS

In a period of turmoil and transformation, the lay *College of Guyenne* in Bordeaux deserves consideration (Gaullieur, 1874). It is based, primarily, on the teachings of the Scottish George Buchanan (1506-1582) (McMillan, 1906; Aitken, 1939) and the Portuguese André de Gouveia (1497-1548), former rector at the University of Paris and then a lecturer in Coimbra (Gouveia, 1583; Woodward, 1924). This bond between France and the Iberian Peninsula will undoubtedly be a *Leitmotif* in the history of European philosophy in the modern era. As Marques de Almeida argues in a valuable study, the sources of Portuguese mathematics in the 14th and 15th centuries are Italian and clearly framed within the tradition of *practical abacus arithmetic*. On careful reading, Marques de Almeida even speaks of hegemony “such an extent that even the Latin versions of Euclid’s *Elements* that circulated in Portugal were due to Campano and Zamberto” (Marques de Almeida, 1995:113)³.

Apart from the foundations of a civic education (Hopkins, 2016) –which would give Renaissance humanism some first-rate figures such as Montaigne (1533-1592) (Frame, 1984)– the *modus parisiensis* was consolidated and expanded at Guyenne. As Danieli argues, it «placed the emphasis above all on the personal activity of the student and proceeded in teaching in a manner that today we would call *seminar-like*» (Danieli, 2014: 47)⁴. This was followed by exercises and tests under the expert guidance of the teacher to consolidate knowledge.

If from a historical, philosophical and pedagogical standpoint, the connections with the Iberian –and in particular the Lusitanian– intellectual tradition can be easily found in the very first Jesuits (O’Malley¹⁹⁹³; Casalini, 2016), the *Society of Jesus* is responsible for the (initially) continental and (then) global success of the *modus parisiensis*. Certainly, in the case of the Jesuit colleges the educational set-up is expression of Ignatian spirituality based on the *Ratio atque institutio studiorum*

³ «Mas foi a tradição italiana, tecida em torno de Pacioli, Cardano, Tartaglia que, entre todas, se tornou dominante. O tempo e o efeito viriam a revelá-la hegemónica de tal modo que mesmo as versões latinas dos Elementos de Euclides que corriam em Portugal deviam-se a Campano e a Zamberto».

⁴ «poneva l’enfasi soprattutto nell’attività personale dello studente e procedeva nell’insegnamento con una modalità che oggi chiameremmo ‘seminariale’».



Societatis Iesu (Pavur, 2006). Its final version is dated 1591 after a series of revisions made in the commission established by the Superior General Claudio Acquaviva (Zardin, 2004).

This commission was initially formed by 12 Jesuits and later replaced in 1584 by a leaner committee of 6 jesuits (Mostaccio, 2014). The *Ratio's* educational method is composed of 463 rules and based on the teaching of Latin and classics. The approach was accompanied by an emphasis on rhetoric and oral expression, based on repetition, mnemonic exercise and dramatic art as well as on Cicero's prose and Greek poetry (Patriarca, 2024). Humanities were for no reason unrelated to in-depth study of Aristotelian philosophy and Thomist (as well as Scotist) theology. On closer inspection, Jesuit pedagogy is also articulated in an original way in terms of the mathematical approach.

A monograph on the influence of the didactic tradition of the abacus in the early pedagogical structure of the Jesuit Colleges has not yet been published, but in a recent study on «*mathematics in the Society of Jesus*» this aspect appears in a no marginal way. According to Bianco, in fact, «the Colleges thus began to compete, locally and otherwise, with these institutes [*abacus schools*], subsuming within a broader framework the teachings of the abacus schools themselves: these Colleges proved preferable, especially for sons of merchants, also in anticipation of an ascent to the rank of nobility»⁵.

But what it is interesting in our case is the primary role played by the curriculum of mathematics built mainly on the *Elements of Euclid* and Jordanus de Nemore's works, through the mediation of Jacques Lefèvre d'Étaples (Pernot, 1995; Oosterhoff, 2018) and Oronce Fine (Axworthy, 2016). In his *Order to follow to attain proficiency in the Mathematical disciplines* (1581), Christopher Clavius (Gatto, 2016), one of the most celebrated Jesuit scientists and student in Coimbra under the guidance of the mathematician Pedro Nunes (1502-1578), repeatedly cites Jordanus de Nemore for arithmetical studies (Casalini & Pavur, 2016: 287). His name is also present in a «second shorter Order for those who are not interested in acquiring a completely thorough understanding of Mathematics» (Casalini & Pavur, 2016: 289).

This aspect is by no means secondary because it demonstrates the reception of some authors even at the non-strictly university level as was already the case in the *abacus schools*. As many studies have shown, moreover, (moral) economy was by no means marginal in Jesuit reflection, which became from time to time more lively and vibrant due to the frequent mobility of members of the *Society of Jesus* and their ability to know and use – *sub specie catholica* – the different cultural and intellectual traditions present in very distant territories (Vismara, 2018). Theology, anyway, stood as a bridgehead for speculation in every sphere of knowledge.

⁵ «I Collegi cominciarono dunque a competere, a livello locale e non, con questi istituti [scuole d'abaco], sussesumendo in un quadro più ampio gli insegnamenti delle stesse scuole d'abaco: tali Collegi si rivelarono preferibili, specie per figli di mercanti, anche in previsione di un'ascesa al rango nobiliare» (Bianco, 2020: 39).



3. THE (ABACUS) CASE OF JUAN DE MARIANA?

Given the nature of this article, one example deserves to be considered (also) as a starting point for future research. Juan de Mariana (1536-1624), one of the most important Jesuits of the *Siglo de Oro* (Laures, 1928), published in 1599 in Toledo a work titled *De Ponderibus et Mensuris* (*On the Weights and Measures*). This book can be included in the abacus tradition especially for the 22 tables with detailed mathematical comparisons (Mariana, 1599: 161-184). It should also be borne in mind that Juan de Mariana had carried out his apostolate in Messina and Rome. In the Sicilian city –at that time part of the transcontinental Spanish Empire– the first Jesuit college (*Primum ac Prototypum Collegium*) was established in 1548 launching an international network of schools, first in Italy and then worldwide (Mattei Casalini, 2014).

The constitutions of that college were drafted by the Spanish Jesuit Geronimo Nadal (1508-1580) (Lasala, 2008: 195-217) and his contribution was so significant that Codina does not hesitate to write that “no one else has probably contributed so much to fixing the Jesuit pedagogical method from its origin. It is not without reason «... that G. Nadal should be rightly considered as the founder of Jesuit pedagogy» (Codina, 1968: XII)⁶. In those years, moreover, the presence in Messina and the great influence of the mathematician Francesco Maurolico (Rose, 1976; Moscheo, 1988 and 1990) –whose interests were wide-ranging in the applied sciences and whose father had also been master of the city mint– cannot be considered marginal (Moscheo, 1998; Sutto, 2000: 59-94; d’Alessandro & Napolitani, 2001: 511-523) .

Returning to Juan de Mariana, in his *Treatise on Weights and Measure*, he dwells –in the typical direct and concise style of abacus treatises– in discussions and definitions of commercial and actuarial themes. Although his writing makes use of the rhetoric and content structure of late scholasticism with its classical references to Greco-Roman philosophy, biblical and patristic sources as well as canonistic apparatus, he demonstrates a thorough knowledge not only of the theological literature applied to commercial disputes but also of the practices in use and its shadowy areas. This is even more evident in his famous *De Monetæ Mutatione* (*On Alteration of Money*) published in 1609 in Cologne.

Two further insights emerge at this point in the line with the education of the merchants. The first related to his homeland, the second to possible connections with the evolution of monetary theories in Italy, that –according to Nussbaum– was at the time «the most advanced country commercially, which had contributed much to monetary practice and thought» (Nussbaum, 1950: 547). Hayek also shared this idea but argued at the beginning of his *Price and Pruduction* –not without a sense

⁶ «Aucun jésuite de son temps n’a aussi profondément connu la Compagnie de Jésus que ce chevalier errant que fut Jerónimo Nadal. Aucun autre n’a probablement contribué autant que lui à fixer dès son origine la méthode pédagogique des jésuites. Ce n’est pas sans motif...que Jerónimo Nadal devrait être considéré à juste titre comme le fondateur de la pédagogie des jésuites».



of irony and sarcasm— that sixteenth-century Italy had «the worst money and the best monetary theory» (Hayek, 1935: 2).

Firstly, the discovery of the New World was followed in Spain by triggering inflationary spirals. As affirmed by Marjorie Grice-Hutchison, «the economic literature of the period reflects the general discontent» of a situation in which «prices had more than doubled» (Grice-Hutchison, 1952: 1). Within this framework, pedagogical and technical reflection evolves across the lines of theology and the tradition of the *ars mercatoria*, undoubtedly supported by mathematical advances. It does not seem, in fact, to be a coincidence that Luis Saravia de la Calle published his *Instrucción de mercaderes (Instruction for Merchants)*, in Medina del Campo in 1544 where moral and technical arguments are treated with special regard to foreign exchange. The same theme is dealt and extended by Cristóbal de Villalón in his *Provechoso tratado de cambios y contrataciones de mercaderes y reprobación de usura*⁷, published in Valladolid in 1546.

Secondly, *De Monetæ Mutatione* (1609) was published approximately twenty years after *Lezione delle Monete* (1588) by Bernardo Davanzati and the treatise *L'Alitinonfo*, the emblematic work by Gasparo Scaruffi published in Reggio Emilia in 1582. In this work—through detailed explanations and mathematical grids typical of the *abacus culture*—a single and universal currency is proposed. Although Mariana «restated Oresme's fundamental argument» (Woodhouse, 2017: 87, f. 8), he uses a technical language typical of the Italian abacus development of the time. Given his long presence in Italy, one might consider a diffusion not only of the *abacus tradition* but also of Scaruffi's work in the Jesuit communities (in which Mariana lived) (Patriarca, 2024, forthcoming).

Recent studies have indeed shown that the Venetian Jesuits had established their own college in Reggio Emilia in 1571 (Salomoni, 2016: 27). At that time in which Milan was under the Spanish domination, this city was a geo-strategical crossroads and an important commercial point between Lombardy, Venice, Tuscany, Genoa and the Papal States. But the even more interesting point is the presence of Bernardino Pratisuoli's commentary on Scaruffi's *L'Alitinonfo* in a catalogue of the Bibliotheca Major of the *Collegio Romano*⁸, currently at the National Central Library in Rome. The copy was in the possession of the Jesuit Giovanni Battista Coccini (1570-1641), Dean of the Auditors of the Apostolic Tribunal of the Roman Rota (De Zaulis, 1672). He decided to bequeath his personal library to the Society of Jesus in 1640 (Breccia Fratadocchi, 2013:53).

⁷ *Profitable treatise on exchanges and merchants' contracts and reprobation of usury.*

⁸ Manuscript Ges. 882 Biblioteca Nazionale Centrale di Roma.

4. PIOUS SCHOOLS AND FREE EDUCATION

If, in Jesuit pedagogy, the influence –albeit mediated– of the *abacus tradition* is directed towards the formation of the ruling classes and the most important families of an expanding merchant bourgeoisie, its contribution does not come to an end but paradoxically and surprisingly finds its survival in other religious movements with educational mission. In this framework, the *abacus pedagogy* becomes a point of reference for those parts of the population that struggled to get out of poverty.

A confirmation is the pedagogical proposal of José de Calasanz Gastón (1557-1648), who during his service in Italy understood its value not only for the innovative didactics but also for social redemption, so much so that he was convinced that «primary education should provide the pupil with some basic knowledge that would be useful for his work and for his life in general. He wanted his students, after school, to be able to make a good living by earning an honest living»⁹.

Founder of the *Pious Schools* in Rome¹⁰ –free schools for the lower classes and vulnerable children– José de Calasanz was born in Peralta de la Sal in the Huesca province of the Kingdom of Aragon in 1557. He studied philosophy and law at the University of Lleida and completed his training in theology in Alcalá de Henares. After a number of ecclesiastical positions in Spain, he moved to Rome in 1592 perhaps with the hope of a prestigious ecclesiastical career (Tosetti, 1824; Tommaseo, 1847; Panchetti, 1977; Santoligi, 1994; Grimaldi, 1997). While in Rome he took the city's poorest and most marginalized youth to heart by founding on November, 27 1597 his first *free public school* (Cremona, 2000). Apart from his friendship with Galileo Galilei and Tommaso Campanella, his pedagogical figure deserves more attention.

While moral education was certainly the basis of his philosophy, no less interesting is the structure of the schools in which general didactics were counterbalanced by an emphasis on hygiene and physical education, by no means primary at that time. Teaching was in Latin and the vernacular, which was not at all considered inferior from a didactic point of view so much so that he defended its use on more than one occasion. This attention to the common language is also evidenced by the production of textbooks in the vernacular and with an essential prose. In this framework, Calasanz institutionalizes the teaching of the abacus within a structured curriculum and expands its centrality to the (international) network of his colleges. In an epistle dated September 11, 1624, defining his educational project and referring to his students, he writes that: «I would like them to have special talent for writing and abacus because they are more valued everywhere and

⁹ «la enseñanza primaria debía ser útil, esto es que aportase al alumno algunos conocimientos básicos que le sirviesen después para su trabajo y para su vida en general. Quería que sus alumnos después de la escuela pudiesen vivir bien ganándose la vida honradamente» (Florensa Parés, 2017: 13).

¹⁰ The *Order of Poor Clerics Regular of the Mother of God of the Pious Schools* founded by Calasanz are commonly known in English-speaking countries as *Piarists*. In Spain they are called *Escolapios*, *Scolopi* in Italian and *Piaristak* in Hungarian.



can be of more benefit to the students, since ordinarily a good writer and abacus-expert attracts many people» (Calasanz, 2017: t. 2, 27, carta 248)¹¹.

This particular penchant for teaching practical and commercial mathematics is also evidenced by the fact that for the preparation of teachers in his schools José de Calasanz did not hesitate to seek the cooperation of his friend Galileo Galilei «and, even after he was condemned by the Inquisition, he encouraged the Florentine Piarists to follow his teaching to the end»¹². Not without difficulty was his acceptance of heliocentrism but also the opposition of private teachers who saw his free schools as ruthless competition (Aporti, 1886: 82). But those teachers would never reach the orphans and the poor to whom, instead, the *Pious Schools* primarily catered. Meanwhile – and this is perhaps the main reason of contrast – the Piarist communities were growing and educating not only according to the standards of the time but also through a very developed vocational training.

5. CONCLUSIONS

History of economic education is at the center of this article, which wished to emphasize the primary importance of the abacus tradition despite the fact that its primacy is not considered at all in many texts on the history of economic thought. The article placed itself on the edge of an educational frontier. This frontier of science and the practical arts –supported by a burgeoning pedagogical awareness– is a source of wonder and cause for admiration.

Three points support this praise. *Firstly*, the abacus schools, undoubtedly, initiated a fundamental process of a broader mathematical literacy with a whole series of positive educational spillovers that would later be used and diluted in (very different) pedagogical systems. *Secondly*, they were fundamental for the creation and institutionalization of an early scientific and technical glossary with its “epistemic and linguistic complexity” (De Mauro, 1994: 413), on which the current language of economics and finance is still based (Dardano, 1998; Sosnowski, 2005). *Thirdly*, the abacus way of reasoning and writing –joining moral urgings from theology and legal disputes over new forms of contracts– gave rise to an entirely original literary genre. In some of these (also widely circulated) books, topics of an ethical and psychological nature are developed along with the classical mercantile themes of accounting and calculus. In these treatises, the human depths and their characteristics were investigated, laying the foundation for the commonly accepted *norms of etiquette* as well as for future *business ethics*.

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¹¹ «Quisiera que ellos tuvieran especial talento para escribir y ábaco porque son más valorados en todas partes y pueden ser de más provecho para los alumnos, pues ordinariamente un buen escritor y abaquista atrae mucha gente».

¹² «e, anche dopo la condanna dell'inquisizione, incoraggiò gli Scolopi fiorentini a seguire fino alla fine il suo insegnamento» (Spinelli, 2001: 128).

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